the right-hand side of the second landing of the public staircase leading from the lower waiting hall up to the Commons Committee Rooms, a brass plate having been fixed upon the wall bearing the following inscription in Elizabethan or church text:—Within this wall are deposited standards of the British Yard Measure and the British Pound Weight, 1853." The certificate is signed by G. B. Airy (Astronomer-Royal), John George Shaw Lefevre (Clerk of the Parliaments), W. H. Miller, C. P. Fortescue (President of the Board of Trade), H. W. Chisholm, and H. J. Chaney; and is dated March 7, 1872.

It hardly appears, therefore, that the old standards of 1758, which appear to have remained unnoticed for the past fifty years, are now of any importance for the purposes of measurement.

MAXWELL'S ELECTRO-MAGNETIC THEORIES.[1]

An account of Maxwell's electric theories from the pen of Prof. Poincaré could not but be full of interest. The volume before us is the first of two on the views and conclusions set forth in the "Electricity and Magnetism" regarding electro-static and electro-magnetic action, and their verification by Hertz and others; and we must of course wait for the completion of the work before we can form any adequate idea of its scope and character, and fully understand the results of the critical analysis which it contains. But in spite of the fact that the treatise is in the somewhat disadvantageous form of an edited course of lectures, it is a contribution of great value to the literature of the subject. Whether or not it is possible always to agree with the physical views expressed regarding matters which are not yet outside the region of speculation, it is impossible not to admire its style and methods. Here are to be found exemplified that order and harmony which render the work of the best French mathematical writers so exquisitely clear, and that artistic charm which is so seldom seen in the writings of scientific men of other nationalities. It has been remarked by competent critics that Maxwell's work, though essentially that of an artist and man of genius, is obscured here and there by a certain vagueness and want of logical coherence and completeness, which has tried the patience and strength of many a devoted disciple. This was of course to a great extent inevitable. He sought out new fields of speculation for himself, and his greatest and most successful generalizations were, one cannot help feeling, the results rather of unerring intuition than of any completely systematic process of reasoning. Those who follow in his footsteps therefore are glad of the help of any friendly guide who is able by his experience and strength to point out the dangers and diminish the difficulties which attend their progress.

In his introduction Prof. Poincaré gives a critical estimate of Maxwell's theories which strikes one at first sight as somewhat inappreciative. Thus he says:—

"La première fois qu'un lecteur français ouvre le livre de Maxwell, un sentiment de malaise, et souvent même de défiance se mêle d'abord à son admiration. Ce n'est qu'après un commerce prolongé et au prix de beaucoup d'efforts, que ce sentiment se dissipe. Quelques esprits éminents le conservent même toujours. . . . Ainsi en ouvrant Maxwell un Français s'attend à y trouver un ensemble théorique aussi logique et aussi précis que l'optique physique fondée sur l'hypothèse de l'éther; il se prépare ainsi une déception que je voudrais éviter au lecteur en l'avertissant tout de suite de ce qu'il doit chercher dans Maxwell et de ce qu'il n'y saurait trouver.

"Maxwell ne donne pas une explication mécanique de l'électricité et du magnétisme; il se borne à démontrer que cette explication est possible.

"Il montre également que les phénomènes optiques ne sont qu'un cas particulier des phénomènes électromagnétiques. De toute théorie de l'électricité on pourra donc déduire immédiatement une théorie de la lumière.

"La réciproque n'est malheureusement pas vraie; d'une explication complète de la lumière, il n'est pas toujours aisé de tirer une explication complète des phénomènes électriques."

The author, however, shows throughout his exposition that he is not only impressed with the extraordinary importance of Maxwell's work, but also thoroughly appreciates and admires, if occasionally under protest and with longing after the more ancient classic models, its somewhat wild and native beauty.

An important part of the introduction is an exposition of the theoretical basis of what Prof. Poincaré rightly regards as the fundamental idea of Maxwell's treatment of electro-magnetism—that is, the application of the general processes of dynamics to any system of current-carrying conductors. No doubt almost all the work which had been done previously had been more or less of this nature, but we refer here to the attempt which Maxwell made with very considerable success to correlate electro-magnetic phenomena by means of Lagrange's general dynamical equations.

In the Lagrangian method the physical state of a system is defined by means of certain parameters $q_1, q_2, \ldots q_n$, n in number; and a dynamical explanation is obtained, or proved to be possible, when the values of these parameters are found in terms of, or proved to be related to the positions and motions of a system of connected particles, either of ordinary matter, or of some hypothetical fluid.

If $m_1, m_2, \ldots m_p$ be the masses of these particles, x_i, y_i, z_i the Cartesian co-ordinates of the particle of mass m_i, and if the system have potential energy V, a function of the $3p$ co-ordinates of type x_i, y_i, z_i, there are $3p$ equations of motion of the form

$$\left.\begin{matrix} m_i\,\ddot{x}_i + dV/dx^i = 0 \\ \&c. \qquad \&c. \end{matrix}\right\} \quad \ldots\ldots \quad (1)$$

The kinetic energy T is

$$\tfrac{1}{2}\Sigma m\,(\dot{x}^2 + \dot{y}^2 + \dot{z}^2),$$

and the principle of conservation of energy gives $T + V =$ constant.

Now we know V, and can express the co-ordinates of each particle or molecule in terms of the n parameters $q_1, q_2, \ldots q_n$. The celebrated Lagrangian equations in terms of the parameters can then be obtained by direct transformation of (1), and are of the type

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_k} - \frac{\partial T}{\partial q_k} + \frac{\partial V}{\partial q_k} = 0.$$

Here T and V are homogeneous quadratic functions, the first of the quantities of type $\dot{q}$, with coefficients which are functions of the parameters themselves, the latter of the parameters only.

If we have reason to believe that the system we are dealing with is a dynamical system, for which the values of T and V (or, more properly, those parts of the total kinetic and potential energies which are concerned in the special phenomenon treated), can be obtained by observation of parameters of type q, we can use these equations in our discussions of results, whether or not we can actually express the parameters in terms of co-ordinates of particles of the system. The justification of this process is the agreement of the results with experiment.

If now we imagine a system of particles (whether of actual or hypothetical matter) say p in number, which has the required values of T and V, and which further gives the same relations of the parameters q, we have obtained a dynamical explanation of the phenomenon. Prof. Poincaré remarks with respect to this process that no dynamical solution of the problem obtained in this way can be unique, and that in fact it must be possible to obtain in this way an infinite number of different solutions, or to quote his own words:—

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[1] "Électricité et Optique." I. Les Théories de Maxwell et la Théorie Électromagnétique de la Lumière. Par H. Poincaré, Membre de l'Institut. (Paris: Georges Carré, 1890.)

"If any phenomenon admits of a complete mechanical explanation it will admit of an infinite number of others which equally well account for all the results of experiment."

This, as he reminds us, is confirmed by the history of physical inquiry. Theories inconsistent with one another are elaborated by different persons, and explain the known facts so well that there is hardly anything left to decide which is right. For example, according to Fresnel the direction of vibration in a ray of plane polarized light is perpendicular to the plane of polarization, according to Neumann and MacCullagh it is *in* the plane of polarization. It can hardly be said that any perfectly absolute *experimentum crucis* has yet been found to decide between these two theories, although the balance of evidence seems decidedly in favour of the view of Fresnel.

It is, however, to be remembered that while we can find different mechanical theories to explain the facts, the theories are not necessarily distinct; the mechanism proposed performs functions which must be performed by the actual mechanism whatever that may be. There always is, as the above cited case well illustrates, a unity connecting the different explanations and a consequent element of similarity among them; and each satisfactory theory elaborated must tend to progress by suggesting modes of deciding in what respects it is redundant or inadequate.

The difficulty then as to real mechanical explanations of phenomena does not prevent us from making progress in our knowledge of matter. The Lagrangian method, and this is its remarkable merit, enables us to use the parameters instead of the co-ordinates of actual particles, and thereby to predict the existence of further properties of matter capable of throwing light on those already observed. In this way may be lightened the task, happily not likely to be soon relinquished by the human intellect, of inquiring into the actual constitution of matter and the mutual actions of its parts.

There seems, however, no doubt that Prof. Poincaré is correct in his view that the central idea of Maxwell's treatise is to prove the existence of a mechanical explanation of electrical phenomena, not indeed actually finding it, but by showing that the Lagrangian method, which presupposes such an explanation, is applicable, and leads to consistent results.

Coming now to the detailed exposition of Maxwell's theories, the first thing that calls for notice is the theory of electric displacement. This has always been a subject of considerable difficulty. What is electricity? is it the ether or something in the ether? in what consists its displacement? are questions which the anxious inquirer is continually putting, and putting in vain. Maxwell's electric displacement and electric force remain simply analogues to the strain and stress in an elastic solid, and it can hardly be said that anyone has yet brought them out of the category of abstractions. No doubt the mechanical analogues suggested by Maxwell himself and by others are helpful in fixing the ideas and enabling the mind to form some concrete conception of what takes place in the medium; but they may easily be carried too far, and prove the means of leading to error. It is almost better in some respects to remain content, if possible, with abstractions, until further light as to the properties of the ether is obtained by experiment and observation; and perhaps it is on this account that Maxwell has abstained from giving such illustrations in his treatise. On the other hand, some notion corresponding to that of electric displacement is necessary for any theory of electrical action regarded as propagated through a medium surrounding the electrified bodies, whose charges become thus the surface manifestation of the state of constraint set up in the dielectric by the electrification.

Prof. Poincaré distinguishes between two fluids—one which he calls *electricity*, and the other the *fluide inducteur*. Both fluids are incompressible, the latter fills all dielectric space, the former is capable of being produced at or placed at any given place or on any given surface. If, then, within a closed space a quantity of electricity is introduced, as, for example, when a charge is placed on the surface of a conductor, an equal quantity of the *fluide inducteur* is forced out across the bounding surface. When all the conductors of a system are in the neutral state, the *fluide inducteur* is in normal equilibrium; when, on the other hand, the conductors are electrified, the equilibrium ceases to be normal and the state becomes one of constraint.

There is some advantage in thus distinguishing between the fluid constituting the electrification and that filling the surrounding space, as it avoids some difficulties of explanation and treatment which arise when only one fluid is considered as producing the phenomena.

After a rather lengthy but in many points critical exposition of the theory of dielectrics, founded on Poisson's notion of *couches de glissement*, we come to an interesting discussion of Maxwell's theory of stresses in a dielectric field. By a somewhat different process from that used by Maxwell, the stresses are found for an isotropic field to be a tension along and a pressure across the lines of force of numerical amount $KF^2/8\pi$, where K is the specific inductive capacity, and F is the electric force at the point considered.

On this result Prof. Poincaré remarks that, although it agrees very well with the observed attractions and repulsions between electrified bodies, yet if these attractions and repulsions are to be considered as due to the existence of such stresses in an elastic medium, the laws of elasticity for that medium must be very different from those for ordinary substances. The ideas of electric displacement and electromotive force at a point correspond to the strain and stress in an elastic solid; but, for correspondence to stresses of the value $F^2/8\pi$, it is necessary to find some different forms of displacement or strain than any that have yet been imagined.

A difficulty here arises to which Poincaré attaches considerable importance. The potential energy in the medium is, if f, g, h be the component electric displacements, given by the equation

$$W = \int \frac{2\pi}{K}(f^2 + g^2 + h^2)dv,$$

where dv is an element of volume and the integral is extended through all space. According to Maxwell's hypothesis as to the localization of the energy of the field, the amount contained in an element dv at which the displacements are f, g, h, is

$$\frac{2\pi}{K}(f^2 + g^2 + h^2)dv,$$

or $KF^2dv/8\pi$. Consequently, if F be increased to $F + dF$, there will be an increase in the potential energy of amount $2KFdFdv/8\pi$. If now the stresses act in the medium as ordinary stresses, they must produce corresponding strains in each element of volume. Hence if the element dv be a rectangular parallelepiped of edges δx, δy, δz when the field is free from electric stress, these dimensions will become, when an electromotive force F is produced at the element, respectively $\delta x\,(1 + e_1)$, $\delta y\,(1 + e_2)$, $\delta z\,(1 + e_3)$. Hence, if when F is increased to F + dF, e_1, e_2, e_3 become $e_1 + de_1$, $e_2 + de_2$, $e_3 + de_3$, the work done by the stresses will, neglecting small quantities of the second order, be

$$\frac{KF^2}{8\pi} dv(de_1 - de_2 - de_3);$$

and if the increase of potential energy in the element take place in consequence of the work done against the stresses we get the equation

$$\frac{F^2}{8\pi} dv(de_1 - de_2 - de_3) = \frac{2FdF}{8\pi} dv,$$

or

$$de_1 - de_2 - de_3 = \frac{2dF}{F},$$

which gives by integration

$$e_1 - e_2 - e_3 = 2 \log F + \text{const.}$$

This result is inadmissible, since when F is zero, we must have $e_1 = e_2 = e_3 = 0$, while if this equation holds either e_2 or e_3 is infinite.

A solution of the difficulty is simply that the energy is not really potential but kinetic. It is certainly not easy to see why the electro-magnetic energy should be regarded as kinetic and the electro-static as potential, and it seems more natural to conclude, as all progress in knowledge of matter seems to indicate, that the properties of the medium are wholly due to motion.

After a short sketch of purely magnetic theory, Poincaré proceeds to what must be regarded as the most important part of his account of Maxwell's work—the theory of electro-magnetism. His investigation of the magnetic potentials of circuits is somewhat different from that usually given. Maxwell takes as his starting point here the equivalence of a current-carrying circuit of small dimensions and a magnet. Poincaré bases his method directly on the following three results of experiment: (1) that two parallel currents of equal intensity and of opposite directions in two close conductors exert no action on a magnetic pole at some distance; (2) if one of these currents have small sinuosities, its action on the magnetic pole is still equal and opposite to that of the straight current; and (3) that the magnetic action is proportional to the quantity of electricity which traverses a cross-section of the conductor in the unit of time.

With the assumption that the components of the force acting on a magnetic pole are obtained by partial differentiation of a function which depends only on the relative positions of the pole and the circuit, the usual theorems are obtained in the following elegant manner. First of all it is shown that the potential of a closed plane circuit at any point in its plane is zero. This is first proved for a circuit symmetrical about a line on its own plane and a point on the axis of symmetry. Then by using the first fundamental proposition to introduce across the circuit straight conductors each carrying two equal and opposite currents equal to the current in the circuit, a circuit of any form is divided into narrow portions each bounded at the ends by elements of the circuit, and at its sides by radial lines passing through the point in question. By using then the second proposition to replace each end-element of the circuit by a circular arc passing through the centre of the element and described from the given point as centre, each strip is turned into a complete circuit, symmetrical about a line through the given point. Since, then, the theorem is true for every such circuit, it is true for the whole given circuit which they build up. Next it is easily shown that when a circuit is situated on the surface of a cone but does not surround the axis—that is, is such that a generating line meets the circuit in an even number of points—the potential of the circuit at the vertex of the cone is zero. For, by means of conductors introduced along generating lines, and carrying equal and opposite currents as before, it is possible with the aid of the second result stated above to replace the circuit by a number of narrow plane circuits each carrying the given current, and symmetrical about a generating line of the cone. Hence each element produces zero potential at the vertex, and therefore so also does the given circuit.

Then it is proved that two circuits on the surface of a cone, each passing round the axis, produce equal and opposite potentials at the vertex, if the currents are equal and flow in opposite directions round the cone. For by means of hypothetical conductors introduced as before along the generating lines, and the second fundamental result, these circuits can be converted into narrow plane circuits, each carrying a current and symmetrical about a generating line. Thus the arrangement of two circuits produces no potential at the vertex. It is to be observed that the two circuits subtend equal solid angles at the vertex of the cone, and that the potentials must still be equal and opposite if the circuits surround distinct superposable cones.

Considering now any closed circuit, we can draw a cone from any chosen point as vertex, so that the generators pass through the circuit. Then this cone can be divided into an infinite number of infinitely small superposable cones of equal solid angle, each having a current flowing round it in the same direction as that round the given circuit, and the total potential at the common vertex is the sum of the equal potentials produced by three small circuits—that is, the potential is proportional to the solid angle subtended at the point by the circuit.

The equations connecting the components u, v, w, of currents with the components of magnetic force and magnetic induction, the relations connecting the magnetic force and magnetic induction, those connecting the magnetic force with the vector potential (which Poincaré calls the *moment électromagnétique*), and the value of the components of the latter quantity for a linear circuit with their application to the proof of Neumann's expression for the "electrodynamic potential" (the mutual intrinsic energy) of two linear current-carrying circuits, and the corresponding expressions for the "electrodynamic potentials" (electrokinetic energies) of the circuits themselves, are dealt with in the next two chapters.

In chapter ix. we come to the most important part of the book, the theory of induction, and the treatment of this part of the subject is instructive. It is a result of experiment that if the currents γ_1, γ_2, in two fixed circuits C_1, C_2, respectively, are varied, electromotive forces $A d\gamma_1/dt + B d\gamma_2/dt$, $B d\gamma_1/dt + C d\gamma_2/dt$ are produced, where B is a coefficient depending on the relative positions of the circuits, A a coefficient depending on C_1 alone, and C a coefficient depending on C_2 alone. Thus if the circuits are deformed or relatively displaced, electromotive forces of amounts $\gamma_1 dA/dt + \gamma_2 dB/dt$, $\gamma_1 dB/dt + \gamma_2 dC/dt$, are produced in C_1 and C_2, so that the total electromotive forces are respectively $d(A\gamma_1 + B\gamma_2)/dt$, and $d(B\gamma_1 + C\gamma_2)dt$. Now by the circuits, in which are supposed to act impressed electromotive forces E_1, E_2, the energy furnished in time dt is $E_1\gamma_1 dt + E_2\gamma_2 dt$. This must be expended in heating the conductors, and in doing *all* the work which is done in the displacement or deformation of the conductors. This latter work is of two parts, (1) that which is done in consequence of the geometrical alteration of the circuits, (2) that which is done in virtue of the change of the current strengths. But the "electrodynamic potential" of the system (Maxwell's electrokinetic energy) is

$$T = \tfrac{1}{2}(L_1\gamma_1^2 + 2M\gamma_1\gamma_2 + L_2\gamma_2^2),$$

so that the former work is

$$\partial T = \tfrac{1}{2}(\gamma_1^2 dL_1 + 2\gamma_1\gamma_2 dM + \gamma_2^2 dL_2).$$

Thus the work dW done in virtue of the changes of the currents is the difference between this and the excess of the energy given out by the batteries over that spent in heat. Thus

$$dW = E_1\gamma_1 dt + E_2\gamma_2 dt - R_1\gamma_1^2 dt - R_2\gamma_2^2 dt - \partial T;$$

and this is the work done in virtue of changes of the currents. This quantity must be a perfect differential, since its integral vanishes for a closed cycle of changes. The condition which must hold for this enables the values of A, B, C to be identified with $-L_1$, $-M$, $-L_2$.

Maxwell's introduction of Lagrange's dynamical method into electro-magnetism is, as has been already stated, regarded by Poincaré as of great importance, and as he says "nous touchons ici à la vraie pensée de Maxwell." After finding by this method the inductive electromotive forces, and the electro-magnetic forces, he proceeds to discuss Maxwell's theorems of the electro-magnetic field, and their crowning generalization, the electro-magnetic theory of light. Except here and there, the treatment differs only in points of detail from that of Maxwell.

With regard to the equations of currents,

$$u = CP + \frac{K}{4\pi}\frac{\partial P}{\partial t},$$
$$\text{\&c.,} \qquad \text{\&c.,}$$

a difficulty is pointed out as to the specific inductive capacity of a conducting substance. For such a substance the first term must preponderate, and so K must be small; whereas K is generally regarded as very great in the case of a conductor. It is worth noticing that this is really only a conventional means of explaining the impossibility of charging a condenser the space between the plates of which is filled with conducting substance; the true explanation is, no doubt, very different.

The discussion of the experimental verifications of the electro-magnetic theory of light contains references to several lately-established experimental facts (apart from Hertz's experiments, which are reserved for special treatment) which bear on the theory. For example, it has been shown by Curie that dielectrics, when tabulated in the order of increasing conductivity, are on the whole arranged (as obviously they should be) in the order of diminishing diathermancy. Further, ebonite, which is opaque to light, is very permeable to dark radiations of longer period, which agrees with its high transparency to electrical waves.

Again, it is remarked that the results of the electro-magnetic theory with regard to reflections from the surface of glass and of metals lend a general support to the theory, while the disagreement in the values of the numerical constants as regards the want of magnetic permeability is referred to the frequency of the vibrations and the fact that the magnetization of the medium is not instantaneously produced.

A marked feature of M. Poincaré's treatise is the chapter on rotatory polarization, in which he discusses the phenomena of rotation of the plane of polarized light by the action of a magnetic field. Although the essential difference between this effect and the apparently similar action of quartz, sugar solutions, &c., is pointed out, the author does not appear to lay stress on it as throwing light on the difference between their causes. For example, after giving Airy's differential equations, for the propagation of the two rectangular component displacements, ξ, η, of a circularly polarized wave travelling along the axis of z, in the form

$$\rho\frac{\partial^2\xi}{\partial t^2} = \frac{\partial^2\xi}{\partial z^2} + a\frac{\partial^3\eta}{\partial z^2\partial t},$$
$$\rho\frac{\partial^2\eta}{\partial t^2} = \frac{\partial^2\eta}{\partial z^2} - a\frac{\partial^3\xi}{\partial z^2\partial t},$$

from which a formula for the rotation of the plane of polarization of plane-polarized light in a magnetic field can be obtained, which agrees with experiment; and after comparing the results of these equations with those of other proposed equations, he says:—

"Mais si le concordance de la formule avec l'expérience justifie l'introduction des derivées $+\partial^3\eta/\partial z^2\partial t$, $-\partial^3\xi/\partial z^2\partial t$ dans les secondes membres des équations du mouvement d'une molécule d'éther, aucune considération théorique ne préside au choix de ces dérivées à l'exclusion des autres; on ne possédait donc pas encore de théorie de la polarisation rotatoire magnétique."

This certainly seems rather too strong a statement in the face of Thomson's dynamical theory outlined in his "Electrostatics and Magnetism," and further elaborated in Maxwell's treatise.

Thomson's views on this subject are of the most fundamental importance, as they point to motion of, or in, the medium occupying the magnetic field as the cause of the magneto-optic effect discovered by Faraday, and to a certain structure of the substance as producing the phenomena shown by quartz, syrup, &c. One of the most interesting passages of his lectures on molecular dynamics, delivered at Baltimore in 1885, is that in which he accounts for the observed results by the presence of rotating particles, "gyrostatic molecules," in the medium.

It is obviously suggested by the gyrostatic investigation that it ought to be possible to explain the magneto-optic rotation in the electro-magnetic theory of light as a consequence of the presence of small magnets embedded in the vibrating medium with their axes in the direction of the ray; and therefore producing a component of magnetization in that direction. It is stated by M. Poincaré that a theory of this kind has been proposed by M. Potier, and published in the *Comptes Rendus*. The theory itself is not given, but the differential equations obtained are quoted, and are of the required form, and lead to the known experimental result.[1]

Maxwell's molecular vortices theory is, however, given, and certain difficulties which it involves discussed. The theoretical results of Hall's experiment are also given in this connection, and Kerr's experiment proving the production of elliptic polarization by the reflection of plane-polarized light from the pole of a magnet is cited, but without any statement of the theory of the effect which has been worked out, principally by Fitzgerald. With regard to the explanation of the Hall effect by strain of the conducting film produced by the magnetic field, it has always seemed to me that it ought to be possible with proper appliances to decide the question, by experimenting with a sufficiently powerful and uniform magnetic field.

The work, it ought to be stated, concludes with an interesting chapter by the editor, M. Blondin, on experimental verifications of the theories of Maxwell. This comprises the chief determinations of specific inductive capacity, Kerr's classical researches, and lastly, the interesting investigation made by M. Röntgen of the electro-magnetic action of currents of displacement.

Of Prof. Poincaré's second treatise on the experiments of Hertz, &c., I hope shortly to give an account as a sequel to the present article.

A. GRAY.

THE ORIGIN OF THE FLORA OF GREENLAND.

HOW the present flora of Greenland originated, is a question of great interest to British botanists and geologists, for the answer will probably help to solve the difficult problem, What was the origin of the recent flora of Britain? The flora of Greenland is so poor in species and has been so well studied that its relationship to the floras of Europe and America ought not to admit of much

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[1] M. Poincaré's reference has suggested to me a mode of investigating the action of these magnets on the electro-magnetic theory. This is discussed in a separate article, which contains for the sake of comparison an account of the gyrostatic theory.